Math 10A
Quiz 8; Friday, 7/27/2018
Time: 3 PM
Instructor: Roy Zhao **Name:** ____________________

Circle True or False. (1 point for correct answer, 0 if incorrect)

1. True **FALSE** It is possible for a BVP to have exactly 2 solutions.

Solution: A BVP can have $0, 1$, or ∞ solutions, not 2.

2. **TRUE** False If we can use both integrating factors and separable equations to solve a differential equation, we must be able to write it in the form $\frac{dy}{dt} = (ay + b)f(t)$.

Solution: It must be separable so it must be of the form $g(y)f(t)$ and since we can use integrating factors, it must be linear in y as well so $g(y)$ must be a linear function or $ay + b$.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (5 points) Find a second order linear homogeneous recurrence relation such that $a_n = 2^n + 3 \cdot 4^n$ is a solution to it.

Solution: Since the bases of the exponents are $\lambda = 2, 4$, the characteristic polynomial is $(\lambda - 2)(\lambda - 4) = \lambda^2 - 6\lambda + 8 = 0$. This corresponds to $a_n - 6a_{n-1} + 8a_{n-2} = 0$ or $a_n = 6a_{n-1} - 8a_{n-2}$.

(b) (5 points) Solve the differential equation $y' = e^t + y$ with $y(0) = 1$.

Solution: We want to use integrating factors because it is linear. It is $y' - y = e^t$. The integrating factor is $I(t) = e^{\int -1 dt} = e^{-t}$. Multiplying through gives us $e^{-t}y' - e^{-t}y = e^t e^{-t} = 1$. Integrating gives us $e^{-t}y = t + C$ so $y = te^t + Ce^t$. Now we plug in the initial condition of $y(0) = 1$ to get $1 = 0e^0 + Ce^0 = C$ so $y = te^t + e^t$.